



University of Kashan

Contents lists available at SCJ

Soft Computing Journal

Journal homepage: <https://scj.kashanu.ac.ir/>

The effective length of vertical rods buried in two-layer soil subjected to lightning return strokes based on ANFIS [◇]

Ali Bagheri¹, MSc student, Sajad Mehrabi¹, MSc student, Saeed Reza Ostadzadeh^{1,*}, Assistant Professor

¹Department of Engineering, Arak University, Arak, Iran.

ARTICLE INFO.

Article history:

Received August 12, 2023

Accepted November 20, 2023

Keywords:

Effective length

ANFIS

Vertical rods

Lightning strokes

Two-layer soil

ABSTRACT

In this paper, an adaptive network fuzzy inference system (ANFIS) based on the Takagi-Sugeno-Kang technique is used for predicting the effective length of vertical rods buried in two-layer soils. The rods are subjected to two typical lightning return stroke currents namely first and subsequent stroke currents. To train the ANFIS approach, a number of input-output pairs are computed from the multi-conductor transmission line method. The inputs are resistivity values of the upper and lower layers, upper layer thickness and the rise time of the lightning current. After the training process is converged, the prediction of effective length is efficiently carried out in such soils. Also, the comparative study with the horizontal electrode buried in two-layer soils shows that the effective length of vertical rods is considerably less than that of the horizontal electrodes which are financially and practically important, whereas in single-layer soil they are different.

2322-3707 / © 2023 The Authors. Open access article under the CC BY license.

1 Introduction

Tower-footing grounding systems such as vertical rods and horizontal electrodes are used to discharge lightning current into the soil. To achieve this aim efficiently, such a device must be designed at an effective length. It is conventionally defined as a starting length at which the slope of impulse impedance (ratio of maxima of transient voltage and injected current) versus the rod length is nil [1]. This definition leads to minimizing the construction cost. Introducing closed-form expression for the effective length is practically of importance. This parameter is strictly dependent on the complex nature of the lossy soil including dispersion [2, 3], ionization [4], and non-homogeneity

[5], separately and simultaneously [6–8]. Hence, researchers proposed formulae based on the curve-fit techniques for single-layer, and dispersive and ionized soils.

The only research on the two-layer soils is related to the harmonic impedance [9, 10], and transient voltage [11]. Fig. 1 shows two conventional grounding systems namely horizontal electrode and vertical rod buried in two-layer soil. In this figure, the upper and lower layer resistivity values and upper layer thickness are given, respectively. These parameters affect the lightning performance of such devices. Recently, Kherif et al. [5], proposed a closed-form expression for the effective length of horizontal electrodes buried in two-layer soil based on combining the numerical solution of the transmission line method (TLM) and genetic algorithm. To the best of our knowledge, there is no closed-form expression for the effective length of vertical rods buried in two-layer soils. This motivates

[◇] Article type: original research

* Corresponding author.

Email addresses: abagheri@msc.araku.ac.ir (A. Bagheri), smehrabi@msc.araku.ac.ir (S. Mehrabi),

s-ostadzadeh@araku.ac.ir (S. R. Ostadzadeh)

the authors to propose efficient formulae for the effective length of vertical rods in such soils based on the adaptive network fuzzy inference system [12]. As reported in the literature, ANFIS is very efficient in comparison with other intelligent methods such as conventional fuzzy inference systems (FIS) [13–15]. During the last ten years, ANFIS has been applied in electromagnetics for instance resonance frequency and radiation resistance of various microstrip antennas [16–19]. Further information about ANFIS in detail is given in the next section.

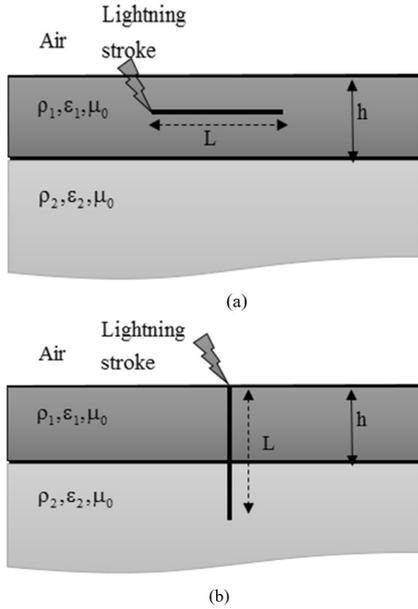


Fig. 1. Two typical grounding systems buried in two-layer soil, (a) horizontal electrode, and (b) vertical rod.

To create the ANFIS approach, a number of input-output pairs are needed in which the inputs are resistivity of upper and lower layers, upper layer thickness, and rise time of the injected current to rod/electrode while the effective length is the output. In this paper, they are computed from the multi-conductor transmission line method (MTL) [20]. The validity of MTL for computing the effective length of grounding electrodes has been recently investigated in [21]. Although this modeling approach is efficient to compute effective length it should be solved iteratively for different values of the rod length up to a starting length at which the slope of impulse impedance versus the rod length is nil.

Also, to include the rise time effect on the effective length, two ANFIS models are proposed separately under two typical lightning currents namely first and subsequent stroke currents. These two lightning currents have low and high rise time values which are conventionally used in analyzing grounding systems under lightning strokes.

The simulation results show that the predicted results based on the ANFIS are in excellent agreement with MTL. The proposed expression makes the design of vertical rods very applicable and avoids repetitive computations when the weather conditions are changed. Also as known in single-layer soil [22], the effective lengths of the vertical rod and horizontal electrode are the same, whereas, they are different in two-layer soils which should be considered by power engineers. The difference between the effective lengths is more pronounced for the first stroke current such that it is less than 70% for low-valued thickness. One of the interesting notes is that the proposed expression can be used for both single and two-layer soils since when the resistivity values of the upper and lower layers are identical, two-layer soil is converted to single-layer soil. Besides, when the upper layer thickness is increased the behavior of single and two-layer soils would be also the same.

This paper is organized as follows. In **Section 2**, modeling principles of MTL and ANFIS approaches are briefly introduced. **Section 3** is focused on the simulation results based on ANFIS and comparison with MTL and the individual ones in horizontal electrodes. Finally, in **Section 4** concluding remarks are presented.

2 Modeling principles

In this section, the modelling principles of MTL and ANFIS respectively as exact and approximate approaches are briefly explained. In both models, it is assumed that two typical lightning currents namely first and subsequent stroke currents as shown in **Fig. 2** are injected into the vertical rod.

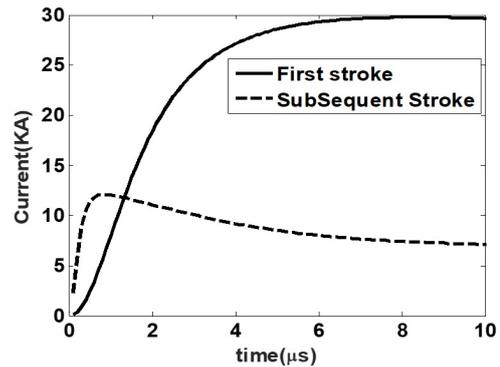


Fig. 2. First and subsequent stroke currents.

The mathematical formula for these two lightning currents is expressed in Eqs. (1) and (2) using Heidler's functions, and its parameters are listed in **Table 1**. Note that the sum of two Heidler's functions is used to represent the subsequent return stroke current. This kind of lightning current is conventionally used for evaluating lightning performance of grounding systems [23, 24].

$$i(t) = \frac{\left(\frac{I_0}{\eta}\right) \exp\left(\frac{-t}{\tau_2}\right) \left(\frac{t}{\tau_1}\right)^n}{\left[1 + \left(\frac{t}{\tau_1}\right)^n\right]} \quad (1)$$

$$\eta = \exp\left[-\left(\frac{\tau_1}{\tau_2}\right) \left(n \frac{\tau_2}{\tau_1}\right)^{\frac{1}{n}}\right] \quad (2)$$

Table 1. Parameters of lightning current adopted from [24].

Current	I_0 (kA)	n	τ_1 (μs)	τ_2 (μs)
First stroke	28	2	1.8	95
Subsequent stroke	10.7	2	0.25	2.5
	6.5	2	2	230

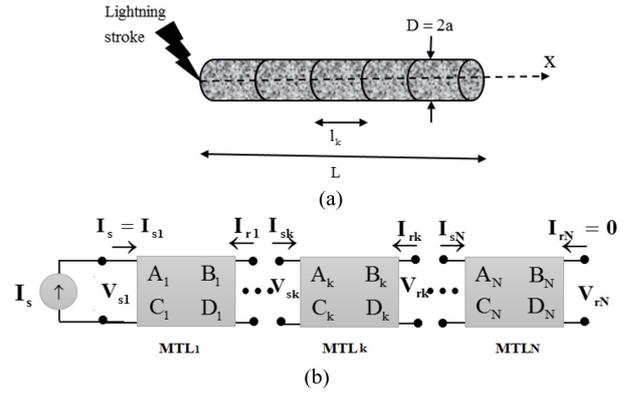
2.1 MTL approach

According to the MTL modeling approach [20], it is naturally a frequency domain method in which each set of parallel conductors is considered as a multi-conductor transmission line (MTL) and connected to each other depending upon the construction of grounding systems. In the special case of vertical rods, a rod of length L is divided into N segments of length $L_k = \frac{L}{N}$, where $k = 1, 2, \dots, N$. The segment length L_k should be satisfied in the relation $L_k < \frac{\lambda}{10}$, where λ is the wavelength. The very short segment improves the accuracy, but the run time increases. Here, we choose $L_k = 0.5m$ resulting in satisfied results. Each segment is then called MTL. The sending and receiving voltages and currents for each segment are connected to each other through an ABCD matrix as follows:

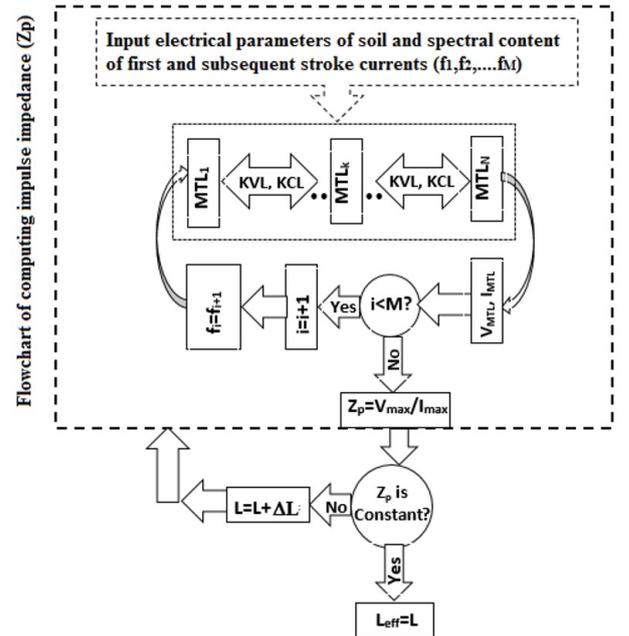
$$\begin{bmatrix} I_{sk} \\ I_{rk} \end{bmatrix} = \begin{bmatrix} A_k & B_k \\ C_k & D_k \end{bmatrix} \begin{bmatrix} V_{sk} \\ V_{rk} \end{bmatrix} \quad (3)$$

where $A_k = D_k = Y_0 \coth(\Psi L_k)$ and $B_k = C_k = -Y_0 \operatorname{csch}(\Psi L_k)$. Y_0 and Ψ are characteristic admittance and propagation constant in the transmission line equations, respectively. Then as shown in Fig. 3, the rod is illustrated as N cascaded MTLs. All MTLs are connected to each other via Kirchhoff's voltage law (KVL) and Kirchhoff's current law (KCL). For instance, the voltages and currents at the connection point of two segments are the same. At the beginning of the first segment, the current (I_s) is the same as the lightning current, whereas at the end of the last segment, the current is nil.

The goal is to compute the sending voltage at the injection point in the time domain. Hence, the spectral content of the two lightning currents should be first extracted at M frequencies f_i , $i = 1, 2, \dots, M$, and MTL equations should be solved at each frequency separately. Applying Inverse Fast Fourier Transform (IFFT) to the ending voltage, results in computing it in time domain (transient voltage) and its maximum is then computed (V_{max}). Finally, the impulse


Fig. 3. Illustrating the vertical rod as N segments and the cascade of MTLs in the frequency domain.

impedance $Z_p = \frac{V_{max}}{I_{max}}$ is easily computed where I_{max} is the maximum value of the lightning current. The mentioned process is iteratively carried out with the length decrement ΔL up to a length at which the slope of impulse impedance versus the rod length is nil, i.e., effective length (L_{eff}). Fig. 4 shows the iteration process of the MTL algorithm for computing transient voltage and accordingly effective length.


Fig. 4. Iteration process in MTL for computing the effective length of a rod.

For the problem under consideration, at first, the spectral contents related to two lightning currents, i.e., first and subsequent stroke currents, are extracted in the time interval of $[0, 10] \mu s$ and tabulated in Table 2. Then, equivalent electrical parameters of two-layer soil [25] are computed at each frequency inside the spectral contents as Eq. (4).

$$\rho_{eq} = \rho_1 \left[\frac{(\sqrt{\rho_2} + \sqrt{\rho_1}) + (\sqrt{\rho_2} - \sqrt{\rho_1})e^{-2h\sqrt{\pi f \mu_0 / \rho_1}}}{(\sqrt{\rho_1} + \sqrt{\rho_2}) - (\sqrt{\rho_2} - \sqrt{\rho_1})e^{-2h\sqrt{\pi f \mu_0 / \rho_1}}} \right]^2 \quad (4)$$

All parameters of Eq. (4), except frequency (f), are illustrated in Fig. 1. Frequency (f) is adopted from Table 2. After that, the MTL approach is applied to each frequency. The mentioned process is repeated for each increased length with a decrement of $\Delta L = 0.5m$. Once the impulse impedance versus the rod length is converged, the starting length in the convergence process is extracted as the effective length.

Table 2. Spectral content of the first and subsequent stroke currents.

Current	First stroke			Subsequent stroke		
Frequency (Hz)	245.7	1018.7	510.7	409	10477	6213
I_s (kA)	551×10^2	760×10^2	391×10^2	8.54	13.62	72.11
	$\angle 30^{deg}$	$\angle 87^{deg}$	$\angle -20^{deg}$	$\angle 39^{deg}$	$\angle 75^{deg}$	$\angle -65^{deg}$

2.2 ANFIS Approach

ANFIS is a class of adaptive networks that are functionally equivalent to fuzzy inference systems [13–15]. The ANFIS architecture consists of five layers including fuzzy layer, product layer, normalized layer, defuzzy layer, and summation layer. A typical architecture of ANFIS for the problem under consideration consisting of three inputs, and a single output is depicted in Fig. 5, in which a circle indicates a fixed node, whereas a square indicates an adaptive node. As seen in this figure, the output of ANFIS is effective length (L_{eff}), while the inputs are the resistivity of upper and lower layers (ρ_1, ρ_2), and upper layer thickness (h).

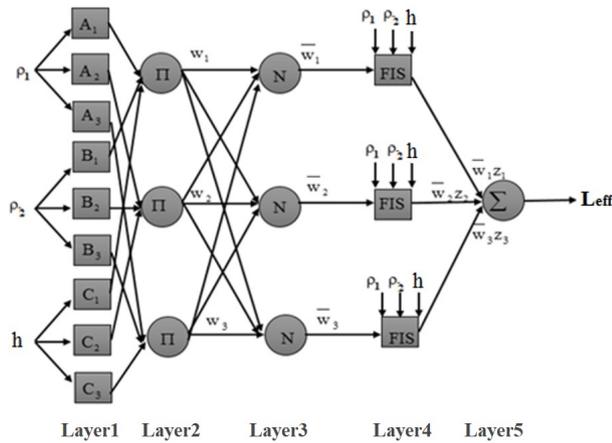


Fig. 5. Schematic of ANFIS approach, where $N = 3$.

In the first layer, the inputs with the use of fuzzy sets are converted to fuzzy inputs with fuzzy values like small, medium, and high. In Fig. 5, fuzzy sets of A_i, B_i, C_i , where $i = 1, 2, \dots, N$, are used for three inputs. The fuzzy sets have belongingness values

between 0 and 1 and are expressed with Gaussian functions as follows:

$$\mu(x) = \exp \left[-\left(\frac{x - c_i}{\sigma_i} \right)^2 \right], \quad i = 1, 2, \dots, N \quad (5)$$

where c_i and σ_i are the center and deviation of the fuzzy sets which are adjusted in the training process using input-output pairs and x is the input variable. For the problem under consideration, each input is expressed with three fuzzy sets.

In the second layer, a weighting factor for each rule is defined as $w_i = \mu_{A_i}(\rho_1)\mu_{B_i}(\rho_2)\mu_{C_i}(h)$, where $i = 1, 2, \dots, N$ and μ_A, μ_B and μ_C are belongingness of the fuzzy sets for inputs.

In the third layer, a normalized weighting factor as defined in Eq. (6) is used.

$$\bar{w}_i = \frac{w_i}{\sum_{i=1}^N w_i} \quad (6)$$

In the fourth layer, the output of FIS is expressed as if-then rules as follows

if (ρ_1 is A_i) and (ρ_2 is B_i) and (h is C_i) **then** $z_i = p_i\rho_1 + q_i\rho_2 + k_i h + r_i$

where $i = 1, 2, \dots, N$. For the problem under consideration, 27 if-then rules are used.

In the last layer, the output is finally computed as follows:

$$L_{eff}(\rho_1, \rho_2, h) = \sum_{i=1}^N \bar{w}_i(p_i\rho_1 + q_i\rho_2 + k_i h + r_i) \quad (7)$$

where the coefficients p_i, q_i, k_i and r_i are computed in the training process based on the least square error technique.

3 Comparative studies

In this section, the proposed models under the first and subsequent stroke currents are evaluated and validated with MTL. Then they are compared with the individual ones in horizontal electrodes [5].

3.1 Comparison with MTL

To create the ANFIS model in this study, $3 \times 3 \times 3 = 27$ input-output pairs are computed using the MTL approach and used in the training process. Each input is expressed linguistically by three fuzzy sets small, medium, and high. The samples for upper and lower layer resistivity and thickness are respectively selected in the intervals of $[100, 1000] \Omega m$, $[100, 1000] \Omega m$, and $[1, 10] m$. Note that for computing effective length, a length decrement $\Delta L = 1m$ is used. Once the training process is converged, the model can be used for predicting the effective length. Fig. 6 shows the root mean square error (RMSE) versus epoch in the training process for the two mentioned currents. The results demonstrate that after 20 epochs these two models converge.

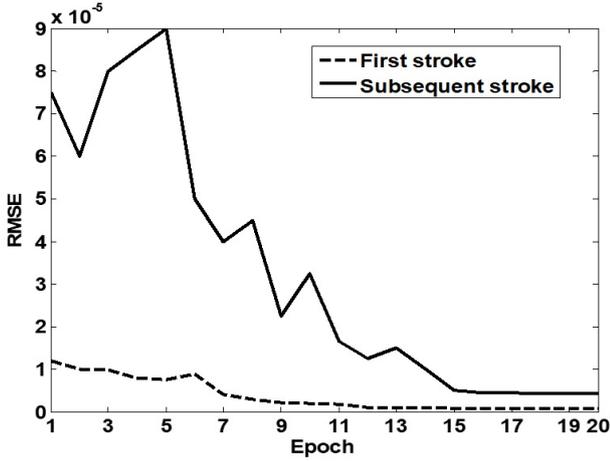


Fig. 6. Root mean square error (RMSE) versus epoch for the first and subsequent stroke currents.

To validate the created models, they are investigated for different scenarios as shown in Fig. 7 and Fig. 8, for the first and subsequent stroke currents, respectively. In the vertical axis of these figures, superscripts “1ST” and “SUB” are denoted for the first and subsequent stroke currents. According to the results, excellent agreement in comparison with the MTL approach is achieved. In addition, the comparison of Fig. 7 and Fig. 8 shows that the same as single-layer soil, the effective length for subsequent stroke current is less than that of first stroke current. It is physically because of the higher frequency content of subsequent current and accordingly more attenuation of the induced electric field inside the soil. Fig. 7 and part (c) of Fig. 8 show that when the upper layer thickness is increased, the effective length is converged to a constant value which is the effective length in single-layer soil. Finally, the approximate run times of different existing methods for computing effective length are compared in Table 3. From this table, ANFIS has the lowest run-time with respect to the other methods which is important from an engineering point of view. Note that the run-time of ANFIS is valid after the training process is converged.

Table 3. Spectral content of the first and subsequent stroke currents.

Current	ANFIS	MTL [5]	TLM [22]
First stroke	0.5 sec.	15 sec.	30 sec.
Subsequent stroke	0.5 sec.	25 sec.	48 sec.

3.2 Comparison with horizontal electrodes

As proven in [23], the effective lengths of vertical rods and horizontal electrodes buried in single-layer soils are identical. Now this issue is investigated in two-layer soils. To this end, the proposed expression in this study is compared with the individual one of horizontal electrodes [5]. The comparison results for both

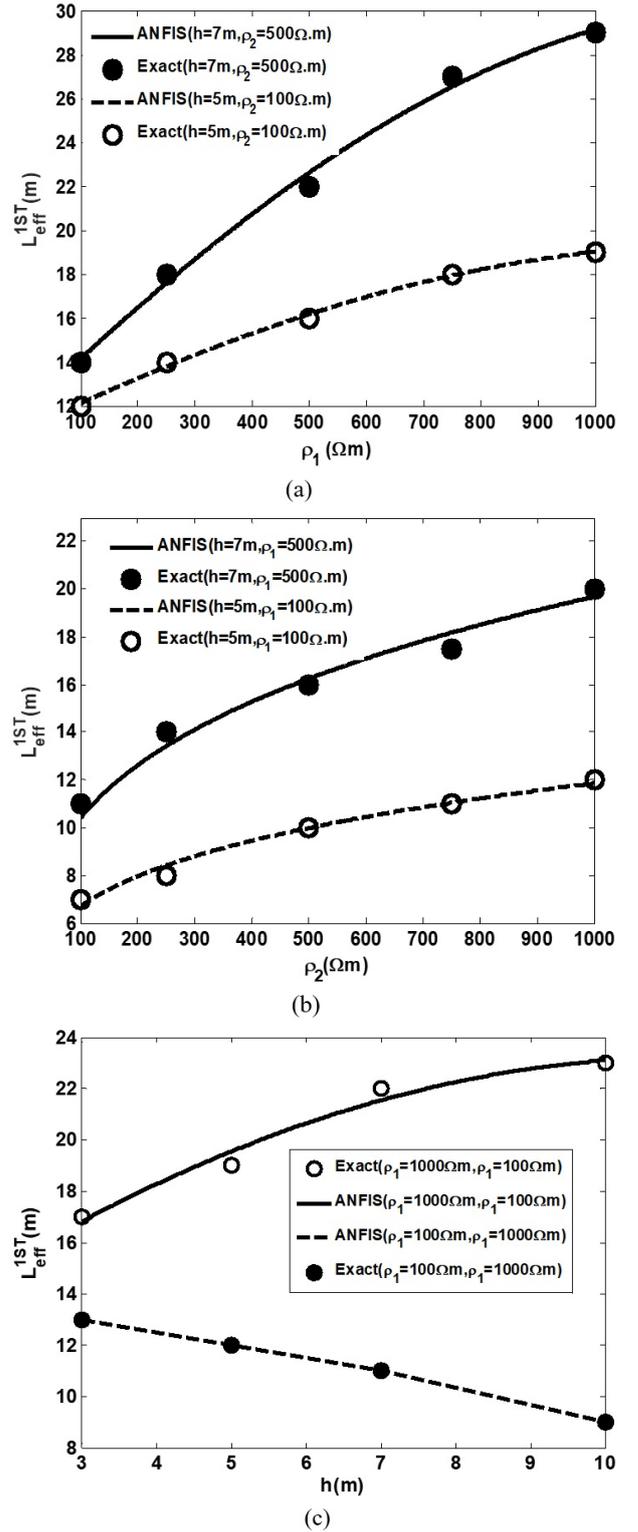


Fig. 7. The effective length of a vertical rod under the first stroke current versus (a): upper layer resistivity (b): lower layer resistivity, and (c) upper layer thickness.

currents are shown in Fig. 9. In this figure, the upper and lower layer resistivity values are respectively 100 Ωm and 1000 Ωm , and the upper layer thickness is varied in the interval of [1, 10] m . The results show that the effective length of the horizontal electrode

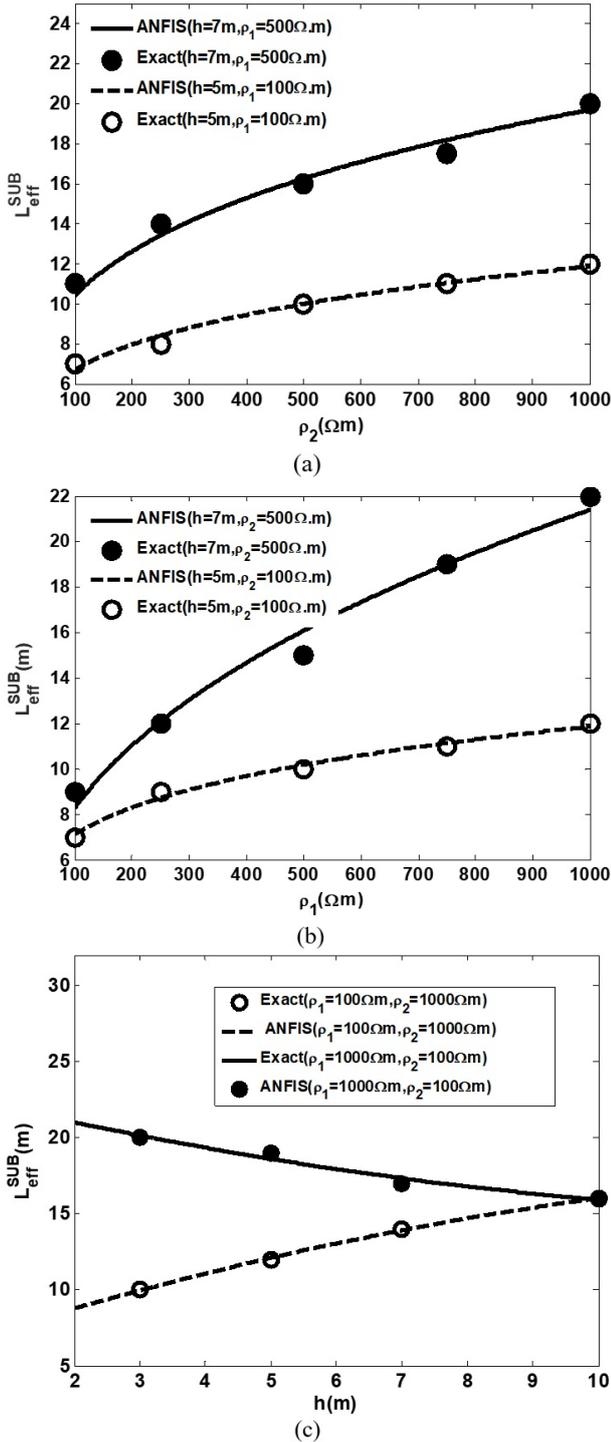


Fig. 8. The effective length of a vertical rod under the subsequent stroke current versus (a): upper layer resistivity (b): lower layer resistivity, and (c) upper layer thickness.

is greater than that of the vertical rod. This difference is more pronounced for the first stroke current such that when the upper layer thickness is 1m, the maximum difference is 69% and 33% for the first and subsequent stroke currents, respectively. On the other hand, when the upper layer thickness is increased, the difference between the effective lengths is decreased.

This is because of converting the two-layer soil into single-layer soil. This conversion is faster carried out for the subsequent stroke current. In the case of subsequent stroke current, when the upper layer thickness is equal to 6 m, both effective lengths are identical. The mentioned finding makes the application of vertical rods more suitable financially in two-layer soils in comparison with horizontal electrodes which should be noticed by power engineers.

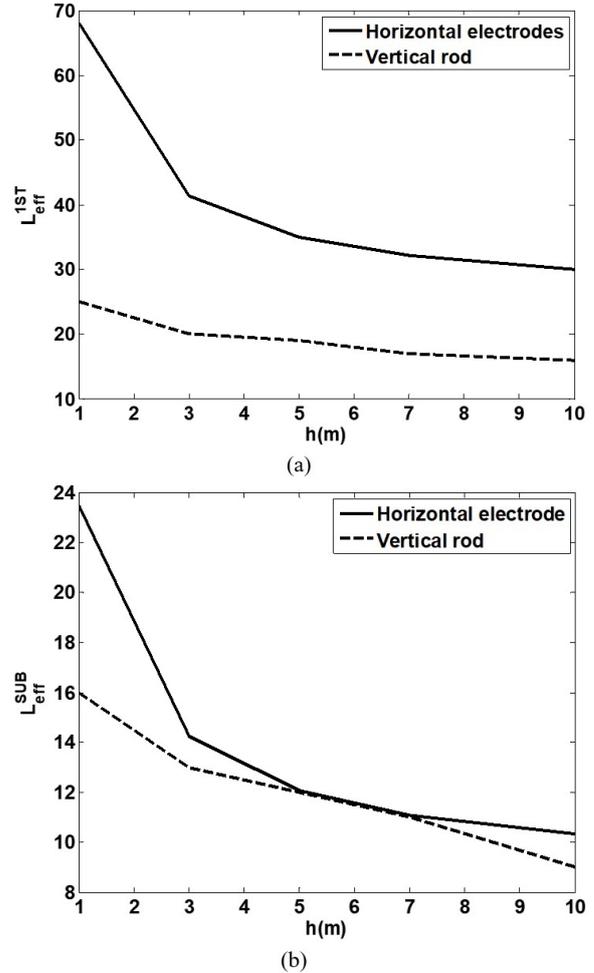


Fig. 9. The effective lengths of vertical rods and horizontal electrodes versus upper layer thickness under (a) the first and (b) subsequent stroke currents.

4 Conclusion

In this paper, a closed-form expression for the effective length of vertical rods under lightning strokes and buried in two-layer soils is extracted. It is based on the adaptive network fuzzy inference system (ANFIS). In extracting the expression, a few input-output pairs are needed which are computed from the multi-conductor transmission line method. Based on the research in this study, the following findings are extracted:

- (1) The proposed expression is very efficient. This makes it suitable from the practical point of

view especially when the climate conditions are changed.

- (2) Due to the proposed expression, predicting the effective length of vertical rods in single and two-layer soils can be simultaneously carried out. Note that when the resistivity values of the upper and lower layers in two-layer soil are the same, the two-layer soil is converted to the single-layer soil.
- (3) Effective lengths of vertical rods and horizontal electrodes buried in two-layer soils are different, whereas in single-layer soils they are the same.

These notes should be considered by power engineers. The proposed approach can be similarly applied to the grounding grid under lightning strikes that are in progress.

Conflict of interest

The authors declare that they have no conflict of interest.

References

- [1] Y. Liu, N. Theethayi, and R. Thottappillil, “Investigating the validity of existing definitions and empirical equations of effective length/area of grounding wire/grid for transient studies,” *J. Electrostat.*, vol. 65, no. 5, pp. 329–335, 2007, doi: 10.1016/j.elstat.2006.09.005.
- [2] R. Alipio and S. Visacro, “Impulse efficiency of grounding electrodes: Effect of frequency-dependent soil parameters,” *IEEE Trans. Power Deliv.*, vol. 29, no. 2, pp. 716–723, 2014, doi: 10.1109/TPWRD.2013.2278817.
- [3] S. Visacro, M. B. Guimarães, and L. S. Araujo, “Experimental impulse response of grounding grids,” *Electr. Power Syst. Res.*, vol. 94, pp. 92–98, 2013, doi: 10.1016/j.epsr.2012.04.011.
- [4] J. He, Y. Gao, R. Zeng, J. Zou, X. Liang, B. Zhang, J. Lee, and S. Chang, “Effective length of counterpoise wire under lightning current,” *IEEE Trans. Power Deliv.*, vol. 20, no. 2, pp. 1585–1591, 2005, doi: 10.1109/TPWRD.2004.838457.
- [5] O. Kherif, S. Chiheb, M. Tegar, A. Mekhaldi, and N. Harid, “Investigation of horizontal ground electrode’s effective length under impulse current,” *IEEE Trans. Electromagn. Compat.*, vol. 61, no. 5, pp. 1515–1523, 2019, doi: 10.1109/TEMPC.2018.2864130.
- [6] S. Kumar A and K. Manickavasagam, “Transmission line dynamic circuit model for effective length of ground electrode under lightning transients,” *IEEE Trans. Electromagn. Compat.*, vol. 64, no. 2, pp. 543–550, 2022, doi: 10.1109/TEMPC.2021.3124679.
- [7] S. S. Sajjadi, V. Aghajani, and S. R. Ostadzadeh, “Comprehensive formulae for effective length of multiple grounding electrodes considering different aspects of soils: Simplified multiconductor transmission line-intelligent water drop approach,” *Int. J. Numer. Model. Electron. Networks, Devices Fields.*, vol. 33, no. 4, p. e2721, 2020, doi: 10.1002/jnm.2721.
- [8] S. S. Sajjadi and S. R. Ostadzadeh, “Predicting formulae for effective length of counterpoise wires buried in ionized, dispersive and inhomogeneous soils,” *COMPEL - Int. J. Comput. Math. Electr. Electron. Eng.*, vol. 39, no. 6, pp. 1375–1391, 2020, doi: 10.1108/COMPEL-08-2019-0327.
- [9] H. Karami, K. Sheshyekani, and F. Rachidi, “Mixed-potential integral equation for full-wave modeling of grounding systems buried in a lossy multilayer stratified ground,” *IEEE Trans. Electromagn. Compat.*, vol. 59, no. 5, pp. 1505–1513, 2017, doi: 10.1109/TEMPC.2017.2655497.
- [10] H. Karami and K. Sheshyekani, “Harmonic impedance of grounding electrodes buried in a horizontally stratified multilayer ground: A full-wave approach,” *IEEE Trans. Electromagn. Compat.*, vol. 60, no. 4, pp. 899–906, 2018, doi: 10.1109/TEMPC.2017.2759259.
- [11] H. Zamani and K. Sheshyekani, “Method of line for modeling of grounding electrodes buried in stratified multilayer soil,” *IEEE Trans. Electromagn. Compat.*, vol. 64, no. 6, pp. 2131–2140, 2022, doi: 10.1109/TEMPC.2022.3210469.
- [12] J. R. Jang, “ANFIS: adaptive-network-based fuzzy inference system,” *IEEE Trans. Syst. Man Cybern.*, vol. 23, no. 3, pp. 665–685, 1993, doi: 10.1109/21.256541.
- [13] H. Abbasi, M. Shamsi, and A. Rasuli Kenari, “Approaches of user activity detection and a new fuzzy logic-based method to determine the risk amount of user unusual activity in the smart home,” *Soft Comput. J.*, vol. 9, no. 2, pp. 2–13, 2021, doi: 10.22052/scj.2021.242812.0 [In Persian].
- [14] H. Moradi-Farahani, J. Asgari, and M. Zekri, “A surveying on type-2 fuzzy logic: Its genesis and its application,” *Soft Comput. J.*, vol. 2, no. 1, pp. 22–43, 2013, doi: 10.1001.1.23223707.1392.2.1.58.2 [In Persian].
- [15] R. Akhoondi and R. Hosseini, “A novel fuzzy-genetic differential evolutionary algorithm for optimization of a fuzzy expert systems applied to heart disease prediction,” *Soft Comput. J.*, vol. 6, no. 2, pp. 32–47, 2018, doi: 10.1001.1.23223707.1396.6.2.3.7 [In Persian].
- [16] K. Guney and N. Sarikaya, “Adaptive neuro-fuzzy inference system for computing the resonant frequency of electrically thin and thick rectangular microstrip antennas,” *Int. J. Electron.*, vol. 94, no. 9, pp. 833–844, 2007, doi:

10.1080/00207210701526317.

- [17] Y. Harkouss, "Accurate modeling and optimization of microwave circuits and devices using adaptive neuro-fuzzy inference system," *Int. J. Microwave Wireless Technol.*, vol. 3, no. 6, pp. 637–645, 2011, doi: 10.1017/S1759078711000651.
- [18] V. Aghajani, S. S. Sajjadi, and S. R. Ostadzadeh, "Design of grounding vertical rods buried in complex soils using radial basis functions," *J. Commun. Eng.*, vol. 7, no. 2, pp. 30–40, 2018, doi: 10.22070/jce.2018.3371.1090.
- [19] K. Guney and N. Sarikaya, "Comparison of mamdani and sugeno fuzzy inference system models for resonant frequency calculation of rectangular microstrip antennas," *Prog. Electromagn. Res. B*, vol. 12, pp. 81–104, 2009, doi: 10.2528/PIERB08121302.
- [20] A. Jardines, J. Guardado, J. Torres, J. Chavez, and M. Hernandez, "A multiconductor transmission line model for grounding grids," *Int. J. Electr. Power Energy Syst.*, vol. 60, pp. 24–33, 2014, doi: 10.1016/j.ijepes.2014.02.022.
- [21] S. R. Ostadzadeh, "Validity of improved mtl for effective length of counterpoise wires under low and high-valued lightning currents," *Adv. Electromagn.*, vol. 9, no. 1, pp. 70–77, 2020, doi: 10.7716/aem.v9i1.1383.
- [22] L. Grcev, "Impulse efficiency of ground electrodes," *IEEE Trans. Power Deliv.*, vol. 24, no. 1, pp. 441–451, 2009, doi: 10.1109/TPWRD.2008.923396.
- [23] M. Akbari, K. Sheshyekani, and M. R. Alemi, "The effect of frequency dependence of soil electrical parameters on the lightning performance of grounding systems," *IEEE Trans. Electromagn. Compat.*, vol. 55, no. 4, pp. 739–746, 2013, doi: 10.1109/TEMC.2012.2222416.
- [24] O. E. Gouda, A. Z. E. D. Mohamed, M. M. Al-Harthi, S. Y. Omar, and S. S. M. Ghoneim, "Performance of grounding electrodes under lightning strokes in uniform and two-layer soils considering soil ionization," *IEEE Access*, vol. 10, pp. 76 855–76 869, 2022, doi: 10.1109/ACCESS.2022.3192394.
- [25] D. A. Tsiamitros, G. K. Papagiannis, and P. S. Dokopoulos, "Homogenous earth approximation of two-layer earth structures: An equivalent resistivity approach," *IEEE Trans. Power Deliv.*, vol. 22, no. 1, pp. 658–666, 2007, doi: 10.1109/TPWRD.2006.881465.