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Sensitivity Analysis of a Mathematical Fuzzy Epidemic Model for Covid-19

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Abstract. In this paper, an epidemic model with fuzzy parameters for spreading Covid-19 in a population is considered. The sensitivity analysis is used to determine the model robustness to parameter values of model. The basic reproduction number of the epidemic model denoted by $\mathcal{R}_0$ determines the dynamics of the model. Then, in order to examine the relative importance of different parameters in the Covid-19 spread, we derive an analytical expression for the sensitivity of the basic reproduction number $\mathcal{R}_0$, namely sensitivity index, with respect to each parameter involved in the model. Finally, sensitivity analysis results and the numerical simulations of the model are given with different parameter values.

Keywords: Fuzzy number, Epidemic model, Covid-19, Basic reproduction number, Sensitivity analysis

1. Introduction

Mathematical models have been widely used to describe the spread of diseases and infections in human populations [1–3]. They help scientists to better understand the effects of intervention strategies on the disease control. The SIR epidemic model has been used as a frame work to investigate Covid-19 infection [4–7]. In [8], authors considered the SIR model (1) and introduced a fuzzy epidemic model by considering parameters of the infection rate, recovery rate, and rate of deaths caused by Covid-19 as fuzzy numbers, and constructing their membership functions.

$$\begin{aligned} \frac{dS}{dt} &= \kappa - \beta(1-\tau)(1-\pi)SI - (\kappa+\tau+\pi)S, \\ \frac{dI}{dt} &= \beta(1-\tau)(1-\pi)SI - (\kappa+\kappa^c+\theta+\gamma)I, \\ \frac{dR}{dt} &= (\theta+\gamma)I + (\pi+\tau)S - \kappa R, \end{aligned} \tag{1}$$

where S, I and R are the number of susceptible, infected and recovered individuals, respectively. β denotes the contact rate, γ is recovery rate, θ is treatment rate, π and τ are proportions of impact of vaccination and compliance with health protocols, κ and κ^c are birth (and also natural death) rate and death rate due to Covid-19, respectively.

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They obtained two equilibria of fuzzy model: the disease-free equilibrium as

$$E^0 = (S^0, I^0, R^0) = \left(\frac{\kappa}{(\kappa + \tau + \pi)}, 0, \frac{\pi + \tau}{\kappa + \tau + \pi} \right),$$

and the endemic equilibrium $E^1 = (S^*, I^*, R^*)$ with

$$\begin{aligned} S^* &= \frac{\kappa + \kappa^c(\eta) + \theta + \gamma(\eta)}{\beta(\eta)(1-\tau)(1-\pi)}, \\ I^* &= \frac{\kappa}{\kappa + \kappa^c + \theta + \gamma} - \frac{\kappa + \tau + \pi}{\beta(1-\tau)(1-\pi)}, \\ R^* &= \frac{(\theta + \gamma(\eta))\bar{I} + (\pi + \tau)\bar{S}}{\kappa}. \end{aligned}$$

Moreover, using the next generation matrix method the fuzzy basic reproduction number was given as

$$\mathcal{R}_0 = \frac{\beta(1-\tau)(1-\pi)\kappa}{(\kappa + \tau + \pi)(\kappa + \kappa^c + \theta + \gamma)},$$

where parameters β, κ^c, and γ are fuzzy numbers. Moreover, by obtaining the eigenvalues of the Jacobian matrix of system (1) at E^0, it was proven that all eigenvalues have negative real part if and only if $\mathcal{R}_0 < 1$ and thus the disease-free equilibrium E^0 is stable if $\mathcal{R}_0 < 1$ and it is unstable if $\mathcal{R}_0 > 1$. In addition, it was shown that the endemic equilibrium E^1 exists and by properties of the second additive compound matrix they concluded that it is stable if $\mathcal{R}_0 > 1$. Thus at $\mathcal{R}_0$ the stability of the system changes and it has a bifurcation. Also, the effect of parameters on $\mathcal{R}_0$ examined via numerical simulations and they concluded that the vaccination and following health protocols have most impact (even more than treatment) on reducing or controlling the infection.

In this paper, we use the sensitivity analysis to discover those parameters that have a high impact on the model studied in [8]. Since the basic reproduction number determines the dynamics of the model, we apply the sensitivity analysis to understand how each parameter influences on the basic reproduction number. Indeed, using this technique we establish an efficient way to reduce $\mathcal{R}_0$ and thus to control the spread of Covid-19.

In the next section we apply sensitivity analysis of the model. In section 3 the simulations of the model are performed and the results obtained by sensitivity analysis are considered numerically. Finally, the results of the paper summarize in section conclusions.

2. Sensitivity analysis

The study of how much uncertainty in each parameter (input) of a variable affects uncertainty in the variable (output) is known as sensitivity analysis. If a variable is a differentiable function of the parameter, the sensitivity index may be alternatively defined using partial derivatives [3, 9, 10].

Definition 2.1. The normalized forward sensitivity index of variable Y that depends differentiably on a parameter x, is defined by $\mathcal{S}_x^Y = \frac{x}{Y} \times \frac{\partial Y}{\partial x}$.

The sign of the sensitivity index $\mathcal{S}_x^Y$ shows the positive or negative impact of the parameter x on variable Y. If $\mathcal{S}_x^Y > 0$ then the value of variable Y increases whenever the value of the parameter x increases and if it is negative then the value of Y decreases whenever the value of the x increases. Also, the magnitude of $\mathcal{S}_x^Y$ shows the increase or decrease amount of Y compared with changes in x.

Since the dynamics of model (1) is determined by the basic reproduction number $\mathcal{R}_0$, we calculate the sensitivity indices for its parameters, which include seven parameters β, τ, π, κ, κ^c, θ and γ. After calculating we get the following normalized forward sensitivity indices for $\mathcal{R}_0$:

$$\mathcal{S}_{\tau}^{\mathcal{R}_0} = \frac{\tau}{1-\tau} \times \frac{-(1+\kappa+\pi)}{\kappa+\tau+\pi} < 0, \qquad \mathcal{S}_{\pi}^{\mathcal{R}_0} = \frac{\pi}{1-\pi} \times \frac{-(1+\kappa+\pi)}{\kappa+\tau+\pi} < 0,$$

$$\mathcal{S}_{\kappa}^{\mathcal{R}_0} = 1-\kappa\Big(\frac{1}{\kappa+\tau+\pi} + \frac{1}{\kappa+\kappa^c+\theta+\gamma}\Big), \qquad \mathcal{S}_{\kappa^c}^{\mathcal{R}_0} = -\frac{\kappa^c}{\kappa+\kappa^c+\theta+\gamma} < 0,$$

$$\mathcal{S}_{\theta}^{\mathcal{R}_0} = -\frac{\theta}{\kappa+\kappa^c+\theta+\gamma} < 0, \qquad \mathcal{S}_{\gamma}^{\mathcal{R}_0} = -\frac{\gamma}{\kappa+\kappa^c+\theta+\gamma} < 0,$$

$$\mathcal{S}_{\beta}^{\mathcal{R}_0} = 1.$$

We see that $\mathcal{S}_{\beta}^{\mathcal{R}_0} > 0$, but $\mathcal{S}_{x}^{\mathcal{R}_0} < 0$ for $x = \tau, \pi, \kappa^c, \theta$ and γ. Also the sign of $\mathcal{S}_{\kappa}^{\mathcal{R}_0}$ is determined after assigning values to parameters. Thus, any increase (or decrease) in values of β yields to increase (or decrease) the value of $\mathcal{R}_0$. However, when the value of any of parameters $\tau, \pi, \kappa^c, \theta$ and γ is increased (or decreased), the value of $\mathcal{R}_0$ will be decreased (or increased).

3. Numerical results

Consider the parameter values in Table 1 for parameters in the model introduced by system (1) and their corresponding sensitivity indices with respect to $\mathcal{R}_0$ as a differentiable function. Sensitivity indices show how a change in each parameter will impact on $\mathcal{R}_0$. For

Table 1. Parameters of the model and their sensitivity index.

Parameter x	Description	Parameter value	$\mathcal{S}_{x}^{\mathcal{R}_0}$
β	contact rate	0.9025	+1
τ	impact of health protocols	0.05	-0.5232
π	impact of vaccination	0.05	-0.5232
κ	birth and natural death rate	0.00625	+0.9283
κ^c	death rate due to infection	0.08004	-0.1643
θ	treatment rate	0.2	-0.4106
γ	recovery rate	0.20083	-0.4123

instance, for these parameter values we have $\mathcal{R}_0 = 0.0984$, while an increasing β by %10, yields also to a %10 increase in $\mathcal{R}_0$, that is, for $\beta = 0.9928$ we get $\mathcal{R}_0 = 0.1082$. While increasing γ by %10, causes a reduction of %4.123 on $\mathcal{R}_0$, that is, for $\gamma = 0.2209$ we find $\mathcal{R}_0 = 0.0945$. Also, from Table 1 we find that vaccination and obeying health protocols have more impact than treatment on reducing $\mathcal{R}_0$.

Now, suppose $\tau = 0$ and $\pi = 0$ and other parameters have values as in Table 1. Considering the unit of time as one day, and initial population as $S(0) = 270000, I(0) = 460, R(0) = 385$ thousand individuals (that are scaled via dividing by total population), the solutions of the model for these sets of parameter values have been simulated using MATLAB software and results have been shown in Figure 1.

4. Conclusion

Applying the sensitivity analysis on the basic reproduction number of the model, the sensitivity indices was found for each parameter. It was shown that $\mathcal{S}_{\beta}^{\mathcal{R}_0} > 0$ and $\mathcal{S}_{x}^{\mathcal{R}_0} < 0$

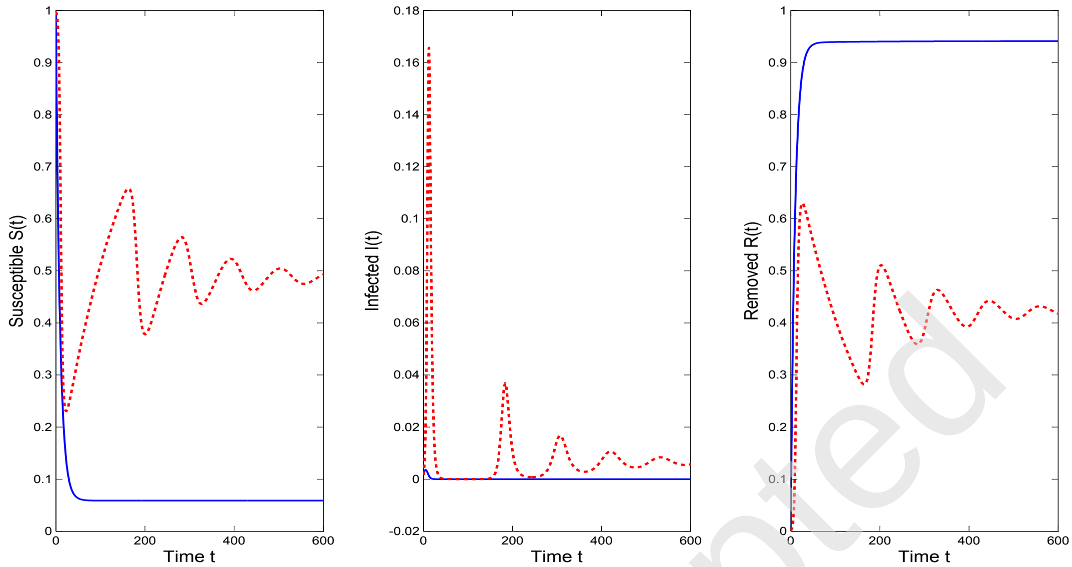

FIGURE 1. Number of susceptible, Infected and recovered individuals for different parameter values. The solid (blue) curves show the solutions of the model for parameter values in Table 1. In this case $\mathcal{R}_0 = 0.0984 < 0$ and thus the Covid-19 infection will be wiped out. The dashed (red) curves are solutions with same parameters except that $\tau = \pi = 0$ which yields to $\mathcal{R}_0 = 2.0529 > 1$. Therefore the infections will remain at a positive level.

for $x = \tau, \pi, \kappa^c, \theta$ and γ. Thus, any decrease in contact rate β yields to decrease in $\mathcal{R}_0$ and causes the control of the Covid-19 infection. Also, any increase in recovery and treatment rates γ and θ decrease the $\mathcal{R}_0$ as well as increase in proportions of vaccination and health protocols and thus lead to a similar result. The results showed that according to the effect of various parameters on the reduction of the basic reproduction number $\mathcal{R}_0$, social distancing (reduction of contact rate), vaccination and adherence to health protocols play a significant role in controlling the infection.

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