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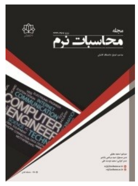

Reduced order framework for 2D heat transfer simulation under variations of boundary conditions based on deep learning algorithms ✧

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Due to the high computational cost of the direct numerical simulation methods of the governing equations of some natural phenomena, surrogate models based on machine learning methods such as deep learning algorithms have been commonly interested in modeling these phenomena. This paper proposes a reduced-order model based on a deep-learning algorithm to simulate temperature changes in a two-dimensional field. This model is developed using three different methods, including a framework based on convolutional neural networks, a physics-informed loss function of the phenomenon, and a reduced-order model using the autoencoder method. The model outcomes were compared with the results obtained from a high-resolution finite difference method. The results show that the reduced-order model (with an accuracy of 2.528×10^{-6} °C) has higher accuracy than the other two models. Meanwhile, the Model-based physics-informed loss is superior to the other two models in terms of steady-state temperature data consumption (only 400 data of size 8×8).

1 Introduction

Partial differential equations are one of the main tools in modeling many phenomena in real life. Some phenomena in nature do not have a definite physical equation and any changes in its behavior cannot be interpreted by a comprehensive law. In addition, many of the governing equations are nonlinear and have complex partial differential equations, so their solution obtained very difficult and usually cannot be solved by mathematical methods. The numerical solution of these equations needs large computation time. In return, deep learning algorithms can instantly simulate complex phenomena without any knowledge of the governing laws [1–3]. Unlike conventional methods, deep-learning models learn to use data-driven methods to generate realistic solutions and greatly reduce the amount of required computation while they have high accuracy. Deep learning algorithms can be used to infer and simulate any dynamic phenomenon by receiving data from observations or simulations. They can also be utilized directly to learn and predict phenomena which are complex or still unknown.

In recent years, many advances in machine vision and natural language processing have been made through deep learning [4–6]. Deep learning methods with the possibility of encrypting appropriate information about the dynamics of the system on the neural network make it possible to teach the neural network how the dynamic system works. Even if it

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is unknown and does not have access to the physical laws of the dynamical system data, it performs modeling of complex systems faster, more accurately and with lower computational cost. According to Andrew Ng, founder, and leader of Google Brain, "Deep learning is like a rocket that its engine is deep learning models, and its fuel are huge amounts of data that are fed to these algorithms" [7].

The present study investigates the simulation of the heat transfer in a two-dimensional field. In the 2D heat transfer problem, we consider a square plate made of some thermally conductive material that is insulated along its edges. Heat is applied to the plate in some way, and our goal is to model the way that thermal energy moves through the plate. The initial condition is given by $T(x, y, 0)$, and we want to determine the temperature field on the plate over time. Under ideal assumptions, it can be shown that temperature satisfies the following two-dimensional heat diffusion equation:

$$\frac{\partial T}{\partial t} = C^2 \nabla^2 T = C^2 \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{1}$$

where $C > 0$ is a constant for the thermal conductivity of the plate. We want to study the solutions that do not vary with time, known as the steady-state solution of the system:

$$\frac{\partial T}{\partial t} = 0 \tag{2}$$

Therefore, the Laplace equation is obtained as follows:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \tag{3}$$

One of the methods that can be used to simulate heat transfer is based on reduced-order models. This method focuses exclusively on the important point of potentially reducing the computational and time costs of simulation [8]. For this purpose, it presents an equation-free reduced order model that uses dimension reduction methods to faster simulation of the steady-state heat transfer.

This research focuses on developing a data-driven model for simulating the two-dimensional steady-state heat transfer according to the variations of boundary conditions. The goal is to introduce and analyze different methods based on deep learning algorithms to be used instead of complex and time-consuming numerical or direct solutions to the governing equations. In this work, these methods are evaluated from broader aspects such as accuracy and data consumption in addition to time and computational cost aspects, and the best and most efficient ones in that aspect are introduced. The methods and models presented in this research have been implemented using the tools and libraries available in Python.

2 Surrogate model based on deep learning algorithms

This section introduces the models based on the deep learning algorithms. These models include a model-based convolutional neural network, a physics-informed loss function of the phenomenon and a reduced-order model using the autoencoder method.

2.1 Model-based convolutional neural network

This method uses a network consisting of multiple convolutional layers, which produce the steady-state temperature distribution by receiving the desired boundary conditions. The training of this network is done in a supervised method, where the data with different boundary conditions are used in the form of a two-dimensional vector as input and the corresponding steady-state temperature data as labels. This model can predict two-dimensional steady-state temperature distribution as a vector.

The network architecture used in this model consists of a two-dimensional convolutional encoder-decoder network adapted from the U-Net architecture in reference [9]. Ten thousand steady-state temperature data in size of 64×64 have been used to train this network.

2.2 Model-based convolutional kernel

The aim of this method is to train a fully convolutional neural network to directly infer the solution to the Laplace equation (Eq. 3), when given the boundary conditions as input [10]. This is accomplished without ever seeing solutions to the boundary problem by encoding the differential equations into a physics-informed loss function, which motivates the main network to find the solution without using supervision in the form of data. The architecture of the network is the same as the U-Net architecture used in the previous model. Table 1 shows the kernel containing the equilibrium conditions pattern obtained from training the network (on 400 data of size 8×8) which is used to train the main network through the loss function defined in Eq. 4.

$$loss = mean(Conv2D(kernel, output))^2 \tag{4}$$

Table 1. The kernel resulting from the training of the network [10].

Result		
3.45×10^{-7}	-2.52×10^{-2}	2.21×10^{-6}
-2.52×10^{-2}	1.01×10^{-1}	-2.52×10^{-2}
3.24×10^{-5}	-2.51×10^{-2}	3.20×10^{-7}

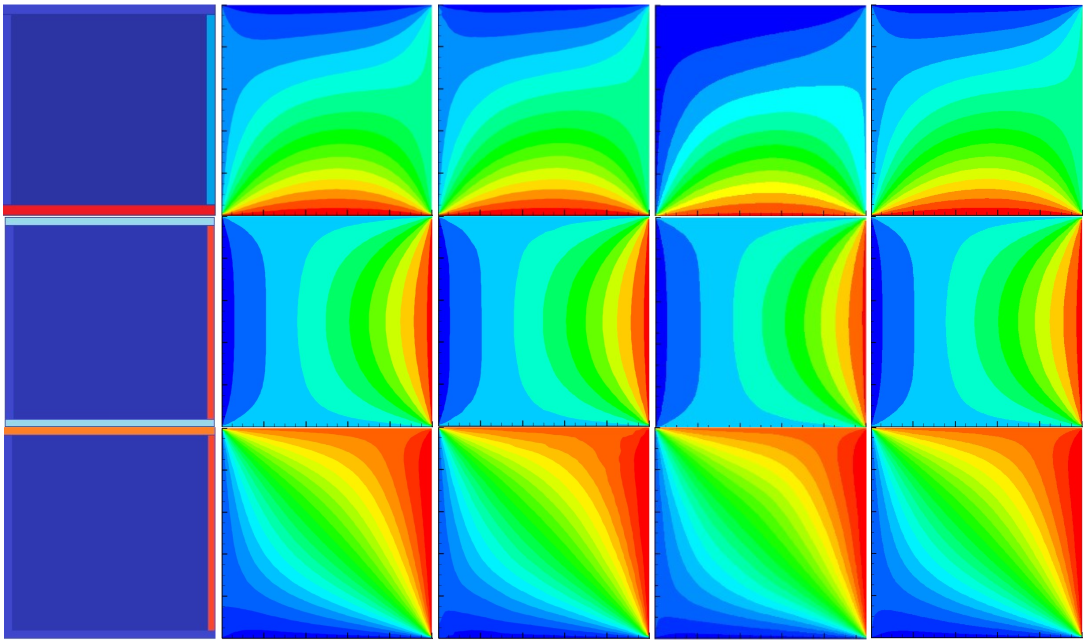

Fig. 1. Results of steady-state temperature distribution using different methods in different boundary conditions in the size of 64×64: (a) control volume with the relevant boundary conditions, (b) correct outputs (finite difference method), (c) model based on convolutional neural network, (d) model-based physics-informed loss function of the phenomenon, and (e) model using the autoencoder method.

2.3 Reduced-order model

In the first step of the model development, the steady-state heat transfer data with different boundary conditions (containing 10,000 data in the size of 64×64) are collected. After normalization and conversion to one-dimensional vectors, it projects on a low-dimensional space using the autoencoder method. We then use different boundary conditions as training data and the data generated with a low dimensional model as a label for training a network consisting of dense layers. This network predicts the steady-state temperature distribution as a reduced vector by receiving the desired boundary conditions in the form of four temperatures related to the 4 faces of the desired control volume. In the next step, in order to obtain the steady-state temperature in the high dimensions, the pattern extracted from the training data is applied to the data generated from the network using the desired dimension reduction method in the opposite direction (to increase the dimensions).

3 Results

Fig. 1 shows the results of steady-state heat transfer modeling in size of 64×64 using different methods. The results demonstrate that the error in terms of the mean squared error for the model based on the convolutional neural network, based on the physics-informed loss function of the phenomenon and reduced-order model using the autoencoder method are equal to 0.015, 0.23 and 2.528×10^{-6} °C per-pixel, respectively.

4 Conclusion

In this paper, three methods based on deep learning algorithms are used to simulate the steady-state heat transfer in size of 64×64. The results are compared with the high-resolution finite difference method (as the exact data). According to the results, reduced order models using the autoencoder method and model-based convolutional neural network have higher modeling accuracy. However, both models used large amounts of data to simulate steady-state heat transfer. In contrast, the model based on the physics-informed loss function of the phenomenon, despite its lower accuracy, requires much less data (only 400 data of size 8×8) to model steady-state heat transfer.

Conflict of interest

The authors declare that they have no conflict of interest.

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